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STUDY ON EXISTENCE OF SOLUTION FOR NON-LINEAR STOCHASTIC DIFFERENTIAL EQUATIONS OF ORDER α ∈ (1,2)

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Abstract:
In this article, we investigate the existence and uniqueness of solution for a class of fractional impulsive stochastic integro-differential equations with finite delay. By using fixed point techniques, we established the existence results and two examples are presented to verified the proposed results.
Keywords: *Fractional differential equation, stochastic differential equations, existence results, impulsive conditions.*

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1 Introduction

During the past decades, modeling with fractional differential equations have become an emerging field of investigation due to their applications in different field of science, engineering, population dynamics, economy and so on. Fractional differential equations have been proved to be an important tool in the modeling of such phenomena as nonlinear oscillations of earthquakes, seepage flow of in porous media. Modeling with derivatives of fractional order also exhibits the memory effect of the system and become an alternative model to integer order differential equations. Theoretical investigation of existence and uniqueness of solutions for such problems did by many authors. Stochastic differential equations are the natural extension of differential equations which also contain some noise (or random) terms and used to model various real world problems such as economic fluctuations, population dynamics and many others areas of engineering and sciences. In some systems the future state of systems not only depends on the present but also on its past which leads to class of stochastic functional differential equations and it has a very important role in various fields such as the population growth model, share price etc. The studies on abstract setting of such equations addressing the issues related to existence, uniqueness and qualitative analysis of solutions, we refer the papers and references therein. It is observed in various dynamical systems in which sudden changes occurs at certain moment of time such changes are naturally seen in mechanics, telecommunications, electronics and economics etc . The impulsive differential equations are more suitable model for describing such effects. For some recent work, in which extensively focused on both impulsive effects and stochastic effects. Wang et al. considered the fractional differential model of order $\alpha \in (1,2)$ with impulsive effects and established a better definition of solutions and builded up a way to find the natural solution of such problems. The authors' proved the sufficient conditions for existence of a unique solution via using fixed point methods. Thereafter, Zhang et al. established the existence and uniqueness of solution of semi-linear impulsive evolution equations with mixed boundary conditions. Recently, Gautam and Dabas studied the existence of solution for the fractional differential model with non instantaneous impulses and in their subsequent study

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in the author's established some more general results. More recently Shu and Shi have developed the right formula of the mild solutions to an impulsive fractional evolution equation and presented the right form of the solutions to the model of linear fractional impulsive evolution equations of order $0 < \alpha < 1$ and $1 < \alpha < 2$, respectively. In case of fractional stochastic differential equations of order (0,1), we have seen sufficient work reported but very limited work is reported for the fractional model of order between (1,2). Yan and Zhang investigated the asymptotical stability of mild solution for nonlinear impulsive fractional neutral partial stochastic differential equations for order between (1,2) and proved the sufficient conditions for the existence and asymptotic stability of solutions by the applications of fixed point methods. Further, Yan et al. extended the studies for a new class of such model with infinite delay and not instantaneous impulses. Motivated by the mentioned work in the papers in this work we discussed the existence and uniqueness of solution for the fractional impulsive stochastic differential equations in the following form:

$$^{c}D_t^{\alpha}x(t) = f(t,x_t,Gx(t)) + g(t,x_t,Gx(t))\frac{dw(t)}{dt}, t \in J = (0,T], t \neq t_k, \quad (1)$$

$$x(t) = \phi(t), x'(t) = \psi(t),\ t \in [-d,0], \quad (2)$$

$$\Delta x(t_k) = I_k(x(t_k^-)), \Delta x'(t_k) = Q_k(x(t_k^-)), k = 1,2,\ldots,m, \quad (3)$$

where cD α t denotes derivative of Caputo's type and here α ∈ (1,2). Here state x(·) gets the value in the separable real Hilbert space H. The functions f : J ×PC0 L ×H → H and g : J×PC0 L ×H → L (K,H) and Ik ,Qk : H → H are appropriate mappings specified later and satisfying certain given conditions and φ(t), ψ(t) are F0- measurable H-valued random variables independent of w. The term

$$Gx(t) = \int_0^t K(t,s)x(s)ds \quad (4)$$

stand for Volterra integral operator. Where K ∈ C(D,R +) is the set of all positive functions which are continuous on D = {(t,s) ∈ R 2 : 0 ≤ s ≤ t < T} and we denote

$$G^* = \sup_{t\in[0,t]} \int_0^t K(t,s)x(s)ds < \infty. \quad (5)$$

To the best of our knowledge, there is no work reported on the existences and uniqueness of solution of the problem in the form (1)-(3). The main aim of this article is to proved the sufficient conditions for the existence and uniqueness results for the class of impulsive fractional stochastic integro-differential equations with finite delay. We established our results by the applications of Banach and Krasnoselskii's fixed point theorems. Rest of this paper is organized as follows. In Section 2, we recall some notations and required preliminaries with proper references. In Section 3, we state and proof the main results and in Section 4, two examples are presented to verifies our claim.

2 Preliminaries

Throughout in this article, we use the symbols (H,k.kH,(.,.)H) and (K,k.kK,(.,.)K) to denote the separable real Hilbert spaces and L (K,H) denotes the space of all bounded linear operators from K into H. The collection of all strongly measurable, square integrable, H-valued random variables, denoted by L 2 (Ω,F,{Ft}t≥0,P;H) = L 2 (Ω;H), is a Banach space equipped with norm kx(·)k 2 L 2 = Ekx(·,w)k 2 H , where E denotes expectation defined by E(h) = R Ω h(w)dP and a subspace is defined as L 2 0 (Ω;H) = { f ∈ L 2 (Ω,H) : f is F0- is measurable}. We assume a Banach space PC0 L = C([−d,0],L 2 (Ω;H)) from [−d,0] into L 2 (Ω;H) of all continuous maps which satisfies supEkφ(t)k 2 < ∞ with norm

$$\|\phi\|_{PC^0_{\mathscr{L}}} = \sup_{t\in[-d,0]} \{E\|\phi(t)\|_{\mathbb{H}}\}, \phi \in PC^0_{\mathscr{L}}.$$

Now we take a Banach space C 1 (J,L 2 (Ω;H)) from J into L 2 (Ω;H) of all continuously differentiable maps which also satisfies the condition supEkx(t)k 2 < ∞ with the following norm

$$\|x\|^2_{C^1} = \sup_{t\in J} \sum_{j=0}^{1} \{E\|x^j(t)\|^2_{\mathbb{H}}\}, x \in C^1(J, \mathscr{L}^2(\Omega;\mathbb{H})).$$

For considering the impulsive effects into the system, we consider a Banach space

$$PC^2_{\mathscr{L}} = PC^1([-d,T], \mathscr{L}^2(\Omega;\mathbb{H})),$$

of all such continuous functions x : [−d,T] → L 2 (Ω;H), which are continuously differentiable on [0,T] except for a finite number of points ti ∈ (0,T), i = 1,2,...,N, at which x ′ (t + i) and x ′ (t − i) = x ′ (ti) exists and endowed with the norm

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$$\|x\|^2_{PC^2_{\mathscr{L}}} = \sup_{t\in J}\sum_{j=0}^{1}\{E\|x^j(t)\|^2_{\mathbb{H}}\}, x \in PC^2_{\mathscr{L}}.$$

To avoid the repetitions of basic definitions of fractional derivatives and integral operator we refer the book [3]. For basics on fractional stochastic differential equations one may see [32]. Lemma 1.An Ft− adapted stochastic process x : [−d,T] → H such that x ∈ PC2 L is said to be a solution of the problem (1)-(3) iff x(0) = φ(0) and x ′ (0) = ψ(0) on [−d,0], and satisfies the following integral equation

$$x(t) = \begin{cases} \phi(0)+\psi(0)t+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s), & t\in(0,t_1], \\ \phi(0)+\psi(0)t+I_1(x(t_1^-))+Q_1(x(t_1^-))(t-t_1)+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds & \\ +\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s), & t\in(t_1,t_2], \\ \dots & \\ \phi(0)+\psi(0)t+\sum_{i=1}^k[I_i(x(t_i^-))+Q_i(x(t_i^-))(t-t_i)]+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds & \\ +\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s), & t\in(t_k,t_{k+1}]. \end{cases} \quad (6)$$

Now, we list the following assumptions.
(H1)The maps f : J ×PC0 L ×H → H and g : J ×PC0 L ×H → L (K,H) are nonlinear, continuous and there exits constants μ1,μ2,η1,η2 such that

$$E\|f(t,\zeta,x)-f(t,\xi,y)\|^2_{\mathbb{H}} \le \mu_1\|\zeta-\xi\|_{PC^0_{\mathscr{L}}}+\mu_2\|x-y\|^2_{\mathbb{H}},$$
$$E\|g(t,\zeta,x)-g(t,\xi,y)\|^2_{\mathbb{H}} \le \eta_1\|\zeta-\xi\|_{PC^0_{\mathscr{L}}}+\eta_2\|x-y\|^2_{\mathbb{H}},$$

for every $x,y\in\mathbb{H}$, $t\in J$ and $\zeta,\xi\in PC^0_{\mathscr{L}}$.

(H2)Ik ,Qk : H → H are continuous functions and there exists constants LI ,LQ > 0, such that

$$E\|I_k(x)-I_k(y)\|^2_{\mathbb{H}} \le L_I E\|x-y\|^2_{\mathbb{H}},$$
$$E\|Q_k(x)-Q_k(y)\|^2_{\mathbb{H}} \le L_Q E\|x-y\|^2_{\mathbb{H}},$$

for every $x,y\in\mathbb{H}$ and $k=1,2,\dots,m$.

(H3)$f: J\times PC^0_{\mathscr{L}}\times\mathbb{H}\to\mathbb{H}$ and $g: J\times PC^0_{\mathscr{L}}\times\mathbb{H}\to\mathscr{L}(\mathbb{K},\mathbb{H})$ are continuous and there exists continuous functions $\mu_1,\mu_2,\eta_1,\eta_2 : J\to(0,\infty)$ such that

$$E\|f(t,v,x)\|^2_{\mathbb{H}} \le \mu_1(t)\|v\|^2_{PC^0_{\mathscr{L}}}+\mu_2(t)\|x\|^2_{\mathbb{H}},$$
$$E\|g(t,v,x)\|^2_{\mathbb{H}} \le \eta_1(t)\|v\|^2_{PC^0_{\mathscr{L}}}+\eta_2(t)\|x\|^2_{\mathbb{H}},$$

where $\mu_1^*=\sup_{s\in[0,t]}\mu_1(s)$, $\mu_2^*=\sup_{s\in[0,t]}\mu_2(s)$, $\eta_1^*=\sup_{s\in[0,t]}\eta_1(s)$, $\eta_2^*=\sup_{s\in[0,t]}\eta_2(s)$ and $t\in J$, $v\in PC^0_{\mathscr{L}}, x\in\mathbb{H}$.

(H4)For the continuous functions I_k,Q_k there exists constants $\rho,\sigma>0$ such that

$$\rho=\max_{1\le k\le m,\ x\in\mathbb{H}}\{E\|I_k(x)\|^2_{\mathbb{H}}\},\ \sigma=\max_{1\le k\le m,\ x\in\mathbb{H}}\{E\|Q_k(x)\|^2_{\mathbb{H}}\}.$$

3 Existence and Uniqueness Results

Theorem 1.Let the assumptions (H1) and (H2) hold and the condition.

$$\Theta=\left\{4(mL_I+mT^2L_Q)+\frac{4T^{2\alpha}}{\Gamma(\alpha)}\left[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*)+\frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)\right]\right\}<1,$$

is satisfied, then the system (1)-(3) has a unique solution.
Proof. We assume an operator N : PC2 L → PC2 L defined by

$$(\mathscr{N}x)(t) = \begin{cases} \phi(0)+\psi(0)t+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s), & t\in(0,t_1], \\ \phi(0)+\psi(0)t+I_1(x(t_1^-))+Q_1(x(t_1^-))(t-t_1)+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds & \\ +\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s), & t\in(t_1,t_2], \\ \dots & \\ \phi(0)+\psi(0)t+\sum_{i=1}^k[I_i(x(t_i^-))+Q_i(x(t_i^-))(t-t_i)]+\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds & \\ +\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s), & t\in(t_k,t_{k+1}]. \end{cases}$$

For showing N a contraction map, we consider two different points x, x ∗ for t ∈ (0,t1] and using the stated assumptions, we have

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$$
\begin{aligned}
E\|(\mathscr{N}x)(t)-(\mathscr{N}x^*)(t)\|_{\mathbb{H}}^2 &\le 2E\|\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}[f(s,x_s,Gx(s))-f(s,x_s^*,Gx^*(s)]ds\|_{\mathbb{H}}^2\\
&+2E\|\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}[g(s,x_s,Gx(s))-g(s,x_s^*,Gx^*(s)]dw(s)\|_{\mathbb{H}}^2,\\
&\le \frac{2T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*)+\frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]\|x-x^*\|_{PC_{\mathscr{L}}^2}^2.
\end{aligned}
$$

In a similar step, when t ∈ (t1,t2], we estimate as

$$
\begin{aligned}
E\|(\mathscr{N}x)(t)-(\mathscr{N}x^*)(t)\|_{\mathbb{H}}^2 &\le 4E\|I_1(x(t_1^-))-I_1(x^*(t_1^-))\|_{\mathbb{H}}^2+4E\|\,Q_1(x(t_1^-))(t-t_1)-Q_1(x^*(t_1^-))(t-t_1)\|_{\mathbb{H}}^2\\
&+4E\|\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}[f(s,x_s,Gx(s))-f(s,x_s^*,Gx^*(s)]ds\|_{\mathbb{H}}^2\\
&+4E\|\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}[g(s,x_s,Gx(s))-g(s,x_s^*,Gx^*(s)]dw(s)\|_{\mathbb{H}}^2,\\
&\le \left\{4(L_I+T^2L_Q)+\frac{4T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*)+\frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]\right\}\|x-x^*\|_{PC_{\mathscr{L}}^2}^2.
\end{aligned}
$$

Similar way for general when t ∈ (tk ,tk+1], k = 2,3,...,m, we obtain

$$
\begin{aligned}
E\|(\mathscr{N}x)(t)-(\mathscr{N}x^*)(t)\|_{\mathbb{H}}^2 &\le \left\{4(mL_I+mT^2L_Q)+\frac{4T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*)+\frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]\right\}\|x-x^*\|_{PC_{\mathscr{L}}^2}^2,\\
&=\Theta\|x-x^*\|_{PC_{\mathscr{L}}^2}^2.
\end{aligned}
$$

Its proved that N is a contraction map, hence there exists a unique x ∈ PC2 L . This completes the proof of the theorem.
Our next existence result is proven by using Krasnoselski's fixed point theorem.
Theorem 2.Let the assumptions (H1),(H3) and (H4) are hold with

$$
\left\{6[\phi(0)+\psi(0)T]+6\rho m+6\sigma mT^2+\frac{6T^{2\alpha}}{\Gamma(\alpha)}[\frac{\mu_1^*+\mu_2^*G^*}{\alpha^2}+\frac{\eta_1^*+\eta_2^*G^*}{T(2\alpha-1}]\right\}\le q,
$$

Where q is a positive real constant, and

$$
\frac{2T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*)+\frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]<1. \tag{7}
$$

Then the problem (1)-(3) has at least one solution on J.
Proof. Let us consider the operators ψ1 : PC2 L → PC2 L and ψ2 : PC2 L → PC2 L defined as

$$
(\psi_1x)(t)=\begin{cases}\phi(0)+\psi(0)t, & t\in(0,t_1],\\ \phi(0)+\psi(0)t+I_1(x(t_1^-))+Q_1(x(t_1^-))(t-t_1), & t\in(t_1,t_2],\\ \cdots & \\ \phi(0)+\psi(0)t+\sum_{i=1}^k[I_i(x(t_i^-))+Q_i(x(t_i^-))(t-t_i)], & t\in(t_k,t_{k+1}],\end{cases}
$$

and

$$
(\psi_2x)(t)=\left\{\tfrac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s,x_s,Gx(s))ds+\tfrac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}g(s,x_s,Gx(s))dw(s),\quad t\in J.\right.
$$

In order to use Krasnoselski's fixed point theorem we will verify that ψ1 is compact and continuous while ψ2 is a contraction operator. For convenience, we combine the proof into 5 steps.
Step 1. In this step, we show that ψ1x+ψ2x ∗ ∈ PC2 L for x,x ∗ ∈ PC2 L .
For t ∈ (0,t1], we have

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$$E\|(\psi_1 x)(t)+(\psi_2 x^*)(t)\|_{\mathbb{H}}^2 \le \{4[\phi(0)+\psi(0)T]+\frac{4T^{2\alpha}}{\Gamma(\alpha)}[\frac{\mu_1^*+\mu_2^*G^*}{\alpha^2}+\frac{\eta_1^*+\eta_2^*G^*}{T(2\alpha-1}]\}\|x\|_{PC_{\mathscr{L}}^2}.$$

Similarly, when t ∈ (tk ,tk+1],k = 1,2,...,m, we get estimate as

$$E\|(\psi_1 x)(t)+(\psi_2 x^*)(t)\|_{\mathbb{H}}^2 \le \{6[\phi(0)+\psi(0)T]+6\rho m+6\sigma mT^2+\frac{6T^{2\alpha}}{\Gamma(\alpha)}[\frac{\mu_1^*+\mu_2^*G^*}{\alpha^2}+\frac{\eta_1^*+\eta_2^*G^*}{T(2\alpha-1}]\}\|x\|_{PC_{\mathscr{L}}^2}$$
$$\le q.$$

This implies that $\|(\psi_1 x)(t)+(\psi_2 x^*)(t)\|_{PC_{\mathscr{L}}^2} \le q$, means $(\psi_1 x)(t)+(\psi_2 x^*)(t) \in PC_{\mathscr{L}}^2$.

Step 2. Secondly, we show that the map ψ_1 is continuous on $PC_{\mathscr{L}}^2$. Let $\{x^n\}_{n=1}^{\infty}$ be a sequence in $PC_{\mathscr{L}}^2$ with $\lim x^n \to x \in PC_{\mathscr{L}}^2$. Then for $t \in (t_k, t_{k+1}], k=0,1,\ldots,m$, we have

$$E\|(\psi_1 x^n)(t)-(\psi_1 x)(t)\|_{\mathbb{H}}^2 \le 2mE\|I_i(x^n(t_i^-))-I_i(x(t_i^-))(t)\|_{\mathbb{H}}^2+2mT^2E\|Q_i(x^n(t_i^-))-Q_i(x(t_i^-))(t)\|_{\mathbb{H}}^2,$$

since the functions $I_i, Q_i, i=1,2,\ldots,m$, are continuous hence $\lim_{n\to\infty} E\|\psi_1 x^n-\psi_1 x\|_{\mathbb{H}}^2=0$ which tells the mapping ψ_1 is continuous on $PC_{\mathscr{L}}^2$.

Step 3. Here we show ψ_1 maps bounded sets into bounded sets in $PC_{\mathscr{L}}^2$.
For this we prove that for $q>0$ there exists $\hat{r}>0$ such that for each $x \in PC_{\mathscr{L}}^2$, we have $E\|(\psi_1 x)(t)\|_{\mathbb{H}}^2 \le \hat{r}$ for $t \in (t_k, t_{k+1}]$, $k=0,1,\ldots,m$. Now, we have

$$E\|(\psi_1 x)(t)\|_{\mathbb{H}}^2 \le 4E\|\phi(0)+\psi(0)t\|_{\mathbb{H}}^2+4E\sum_{i=1}^{\kappa}\|I_i(x(t_i^-))\|_{\mathbb{H}}^2+4E\sum_{i=1}^{\kappa}\|(t-t_i)Q_i(x(t_i^-))\|_{\mathbb{H}}^2,$$
$$\le 4(\phi(0)+\psi(0)T)+4m\rho+4mT^2\sigma=\hat{r}$$

which proves the required result. Step 4. The map ψ1 is equicontinuous. Let u,v ∈ (tk ,tk+1],tk ≤ u < v ≤ tk+1, k = 0,1,2,...,m,x ∈ PC2 L we obtain

$$E\|(\psi_1 x)(v)-(\psi_1 x)(u)\|_{\mathbb{H}}^2 \le 2E\|(v-u)\psi(0)\|_{\mathbb{H}}^2+2m\|(v-t_i)-(u-t_i)\|^2E\|Q_i(x(t_i^-))\|_{\mathbb{H}}^2.$$

As v → u, then limu→v Ek(ψ1x)(v)−(ψ1x)(u)k 2 H = 0, which implies that ψ1 is equicontinuous. Finally, combining Step 1 to Step 4 together with Ascoli's theorem, we conclude that the operator ψ1 is compact. Step 5. In this step, we prove that the map ψ2 is a contraction map. Let x,x ∗ ∈ PC2 L and t ∈ (tk ,tk+1], k = 0,1,...,m, we get

$$E\|(\psi_2 x)(t)-(\psi_2 x^*)(t)\|_{\mathbb{H}}^2 \le 2E\|\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[f(s,x_s,Gx(s))-f(s,x_s^*,Gx^*(s)]ds\|_{\mathbb{H}}^2$$
$$+2E\|\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[g(s,x_s,Gx(s))-g(s,x_s^*,Gx^*(s)]dw(s)\|_{\mathbb{H}}^2,$$
$$\le \frac{2T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*)+\frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]\|x-x^*\|_{PC_{\mathscr{L}}^2}.$$

Hence ψ2 is a contraction map. Therefore, we deduce that the problem (1)-(3) has at least one solution on J. This completes the proof of the theorem.

4 Examples

4.1 Example

Let us investigate the following fractional stochastic problem

$$D^{\alpha}u(t)=\frac{u(t-d)}{25+u(t-d)}+\int_0^t k(s,t)e^{-u(s)}ds+\left[\frac{u(t-d)}{36+u(t-d)}+\int_0^t k(s,t)e^{-u(s)}ds\right]\frac{d\beta(t)}{dt}, \quad (8)$$

$$t\in[0,1],\ x\in(0,\pi),\ t\neq\frac{1}{2}. \quad (9)$$

$$u(t)=\phi(t),\ u'(0)=0,\ t\in[-d,0], \quad (10)$$

$$\Delta u|_{t=\frac{1}{2}^-}=\sin(\frac{1}{49}\|u(\frac{1}{2}^-)\|);\ \Delta' u|_{t=\frac{1}{2}^-}=\cos(\frac{1}{49}\|u(\frac{1}{2}^-)\|), \quad (11)$$

where D α is fractional derivative and is taken in Caputo's sense for α ∈ (1,2), 0 < t1 < 1 are prefixed numbers and φ ∈

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PCL 2 . Now it is easy to see that

$$E\|f(t,\phi,x)-f(t,\psi,y)\|^2_{\mathbb{H}} \le \frac{1}{25}\|\phi-\psi\|^2_{PC^2_{\mathscr{L}}} + E\|x-y\|^2_{\mathbb{H}},$$

$$E\|g(t,\phi,x)-g(t,\psi,y)\|^2_{\mathbb{H}} \le \frac{1}{36}\|\phi-\psi\|^2_{PC^2_{\mathscr{L}}} + E\|x-y\|^2_{\mathbb{H}},$$

$$E\|I_k(x(t^-))-I_k(y(t^-))\|^2_{\mathbb{H}} \le \frac{1}{49}E\|x-y\|^2_{\mathbb{H}},$$

$$E\|Q_k(x(t^-))-Q_k(y(t^-))\|^2_{\mathbb{H}} \le \frac{1}{49}E\|x-y\|^2_{\mathbb{H}},$$

then with the above said settings the problem (8)-(11) can be rewritten in the abstract form of the equations (1)-(3). Further more, we have μ1 = 1 25 , μ2 = 1, η1 = 1 36 , η2 = 1, LI = 1 49 , LQ = 1 49 , m = 1, T = 1,α = 3 2 , G ∗ = 1 20 , put these values in the condition given in the Theorem 1 as

$$\Theta = \left\{4(mL_I + mT^2L_Q) + \frac{4T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*) + \frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]\right\},$$

we get $\Theta = .52 < 1$, which implies that problem (8)-(11) has a unique solution in $[0,1]$.

4.2 Example

Here, we consider the following impulsive fractional stochastic differential model

$$^CD^\alpha z(t) = \frac{e^t z(s-d)}{36} + \int_0^t k(t,s)\sin z(t-d)ds + [\frac{e^{2t}z(s-d)}{108} + \int_0^t k(t,s)\sin z(t-d)ds]\frac{dw(s)}{ds}, \quad (12)$$

$$z(t) = \phi(t),\ z'(t) = \rho(t),\ t\in[-d,0], \quad (13)$$

$$\Delta z(t_i) = \int_0^\pi \frac{\sigma_i(s,z(t_i))}{81}ds,\ \Delta z'(t_i) = \int_0^\pi \frac{\xi_i(s,z(t_i))}{81}ds, \quad (14)$$

where $^CD^\alpha$ is fractional derivative with order $\alpha \in (1,2)$, of Caputo type and $0 < t_1,\cdots,t_m < t_{m+1} = 1$ are prefixed numbers. If we put $t-d = \theta$ for $\theta \in [-d,0]$ then on rearranging the terms as

$$f(t,\varphi,u) = \frac{e^t\varphi}{36} + \int_0^t k(t,s)\sin\varphi ds,$$

$$g(t,\varphi,u) = \frac{e^{2t}\varphi}{108} + \int_0^t k(t,s)\sin\varphi ds$$

$$I_k(z) = \int_0^\pi \frac{\sigma_i(s,z(t_i))}{81}ds,$$

$$Q_k(z) = \int_0^\pi \frac{\xi_i(s,z(t_i))}{81}ds.$$

From the above functions we set the following values as

$$E\|f(t,\varphi,u)-f(t,\psi,v)\|^2_{\mathbb{H}} \le \frac{1}{36}\|\varphi-\psi\|^2_{PC^2_{\mathscr{L}}} + E\|u-v\|^2_{\mathbb{H}},$$

$$E\|g(t,\varphi,u)-g(t,\psi,v)\|^2_{\mathbb{H}} \le \frac{1}{108}\|\varphi-\psi\|^2_{PC^2_{\mathscr{L}}} + E\|u-v\|^2_{\mathbb{H}},$$

$$E\|I_k(z(t^-))-I_k(y(t^-))\|^2_{\mathbb{H}} \le \frac{\sigma*}{81}E\|z-y\|^2_{\mathbb{H}},$$

$$E\|Q_k(z(t^-))-Q_k(y(t^-))\|^2_{\mathbb{H}} \le \frac{\xi*}{81}E\|z-y\|^2_{\mathbb{H}},$$

Then with these settings the problem (8)-(11) can be rewritten in the abstract form of the equations (1)-(3). Furthermore, we consider the values as σ∗ = ξ ∗ = 1,µ1 = 1 12 , µ2 = 1, η1 = 1 18 , η2 = 1, LI = 1 81 , LQ = 1 81 , m = 1, T = 1,α = 3 2 , G ∗ = 1 49 , and estimate the condition given in the Theorem 1 as

$$\Theta = \left\{4(mL_I + mT^2L_Q) + \frac{4T^{2\alpha}}{\Gamma(\alpha)}[\frac{1}{\alpha^2}(\mu_1+\mu_2G^*) + \frac{1}{T(2\alpha-1)}(\eta_1+\eta_2G^*)]\right\},$$

we get Θ = .47 < 1, hence the problem (8)-(11) has a unique solution in [0,1].

5 Conclusions

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We have established the existence and uniqueness of solution for the problem (1)-(3). We proved our results by combining the theory of stochastic analysis with fixed point methods. The existence and uniqueness of solutions for the system (1)-(3) deals with the help of Banachs and Krasnoselskiis fixed point theorems. Our future work will be to extend these results for the higher order derivatives problems with not instantaneous impulses.

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